Computation of induced fields into the human body by using dual formulations

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***Abstract*—In this work we present the use of two dual formulations to compute Extremely Low frequency (ELF) induced fields into the human body. This allows to estimate the numerical error, as well as rigorously bound the (global) co-energy. This method is here applied to the classical case of the exposure to a homogeneous magnetic field.**

I. INTRODUCTION

Numerical dosimetry of Extremely Low Frequency (ELF) fields induced in the human body is important in order to better tune and/or understand the action of recent medical devices which make use of electrical energy [1], and for limiting human exposure to electromagnetic fields [2]. Due to the difficulty of obtaining quality meshes from segmented images, most computations are performed by means of Finite Difference (FD) methods [3] as they are straightforwardly applied to "hexaedric" meshes. Despite the effort of the scientific community, a convincing validation of numerical simulation is still lacking [4]. This is particularly annoying due to the extreme complexity of the human body, and to the uncertainties on dielectric properties of tissues [5]. Therefore it is crucial to reduce as much as possible the numerical errors.

II. FORMULATIONS FOR NUMERICAL DOSIMETRY

In the quasi-static approximation, Maxwell's equations for a magnetodynamic problem can be simplified as:

$$\text{curl } \mathbf{e} = -\partial_t \mathbf{b}\ , \quad \text{curl } \mathbf{h} = \mathbf{j}\ , \quad \text{div } \mathbf{b} = 0\ , \qquad (1\,\text{a,b,c})$$

$$\mathbf{j} = \sigma \mathbf{e}\ , \quad \mathbf{b} = \mu \mathbf{h}\ , \qquad (1\,\text{d,e})$$

with $\mathbf{e}$ the electric field, $\mathbf{h}$ the magnetic field, $\mathbf{j}$ the electric current density, $\mathbf{b}$ the magnetic flux density, σ the electrical conductivity and μ the magnetic permeability. Displacement currents and the reaction field are neglected [6]: this allows to reduce the computational domain Ω to the human body, on the boundary of which the following condition has to be imposed:

$$\mathbf{n} \cdot \mathbf{j}|_{\partial\Omega} = 0 \qquad (2)$$

Furthermore, as the reaction field is disregarded, one reckons that indeed equations (1a) – (1e) have the mathematical structure of a static problem.

At the continuous level, these fields are welcome into the structure represented in the following Tonti's diagram:

$$\begin{array}{ccccccc} \phi & \xrightarrow{\text{grad}} & \mathbf{e}, \mathbf{a} & \xrightarrow{\text{curl}} & \mathbf{b} & \xrightarrow{\text{div}} & 0 \\ & & \sigma \updownarrow & & \mu \updownarrow & & \\ 0 & \xleftarrow{\text{div}} & \mathbf{j} & \xleftarrow{\text{curl}} & \mathbf{h}, \mathbf{t} & & \end{array} \qquad (3)$$

R. Scorretti thanks the ACE, University of Liège for the invitation which allowed doing this work.

At the discrete level this structure is approximated by appropriate mixed Finite Elements (FE). Two dual formulations are obtained by strongly imposing the constitutive laws and the equation on one of the two levels of Tonti's diagram, whereas the equation on the other level is weakly imposed [7].

A. *The e–conform* $\phi - \mathbf{a}$ *formulation*

Let $\mathbf{a}$ be a *known* magnetic vector potential such that: curl $\mathbf{a} = \mathbf{b}$. By enforcing in a strong sens the upper level of (3), i.e. Faraday's law (1a) one obtains that: $\mathbf{e} = -\partial_t \mathbf{a} - \text{grad } \phi$, where ϕ is an unknown electric scalar potential. The weak form of Ampère's law (1b) reads [6]:

$$(\sigma(\partial_t \mathbf{a} + \text{grad } \phi),\ \text{grad } \phi') = 0 \quad \forall \phi' \in \text{H}(\text{grad}, \Omega), \quad (4)$$

where $(\cdot,\cdot)$ denotes a volume integral in Ω of the product of vector fields.

B. *The j–conform* $\mathbf{t} - \mathbf{b}$ *formulation*

Analogously, we can strongly enforce the lower level of (3), i.e. the divergence of Ampère's law (1b), div $\mathbf{j} = 0$. Let $\mathbf{t}$ be an unknown electric vector potential such that: curl $\mathbf{t} = \mathbf{j}$. The weak form of Faraday's law (1a) is given by [8]:

$$(\frac{1}{\sigma}\text{curl } \mathbf{t},\ \text{curl } \mathbf{t}') + (\partial_t \mathbf{b},\ \mathbf{t}') = 0 \quad \forall \mathbf{t}' \in \mathbf{H}_0(\text{curl}, \Omega). \quad (5)$$

However, this formulation gives rise to a linear system which is difficult to solve when the imposed flux density $\mathbf{b}$ is not exactly solenoidal. It has been shown [9] that this problem can be solved by projecting $\mathbf{b}$ on the kernel of the div operator $\mathbf{H}(\text{div } 0, \Omega)$. That is, a vector potential $\mathbf{a}$ such that $\mathbf{b} = \text{curl } \mathbf{a}$ is computed and (6) becomes:

$$(\frac{1}{\sigma}\text{curl } \mathbf{t},\ \text{curl } \mathbf{t}') + (\partial_t \mathbf{a},\ \text{curl } \mathbf{t}') = 0 \quad \forall \mathbf{t}' \in \mathbf{H}_0(\text{curl}, \Omega) \quad (6)$$

III. ERROR ESTIMATE AND CO-ENERGY BOUND

Considering the dual electromagnetic formulations together allows, on the one hand, to calculate a more precise solution as the average of the dual solutions:

$$\mathbf{e} = \frac{1}{2}\left(\mathbf{e}_1 + \frac{1}{\sigma}\mathbf{j}_2\right) = \frac{1}{2}\left(-\partial_t \mathbf{a} - \text{grad } \phi + \frac{1}{\sigma}\text{curl } \mathbf{t}\right), \quad (7)$$

where $\mathbf{e}_1$ and $\mathbf{j}_2$ are computed respectively with the $\phi - \mathbf{a}$ and $\mathbf{t} - \mathbf{b}$ conform formulation. On the other hand, the difference: $\Delta\mathbf{e} = ||\mathbf{e}_1 - \frac{1}{\sigma}\mathbf{j}_2||$ is an estimate of the numerical error.

A rigorous bound can be found for the coenergy $\mathcal{E}^C$, defined at the continuous level:

$$\mathcal{E}^C = \int_\Omega \int_0^{\mathbf{e}} \mathbf{j}\,\mathrm{d}\mathbf{e} \tag{8}$$

At the discrete level, by applying $\mathrm{div}\ \mathbf{j}_2 = 0$ strongly, dual coenergies can be computed:

$$\mathcal{E}_1^C = \int_\Omega \int_0^{\mathbf{e}_1} \mathbf{j}\,\mathrm{d}\mathbf{e} = \frac{1}{2}(\sigma\mathbf{e}_1,\ \mathbf{e}_1) \tag{9}$$

$$\mathcal{E}_2^C = \int_\Omega \mathbf{e}_1 \cdot \mathbf{j}_2 - \int_\Omega \int_0^{\mathbf{j}_2} \mathbf{e}\,\mathrm{d}\mathbf{j} = \frac{1}{2}(-\partial_t \mathbf{a},\ \mathbf{j}_2) - \frac{1}{2}\Big(\frac{1}{\sigma}\mathbf{j}_2,\ \mathbf{j}_2\Big) \tag{10}$$

By extending a result from static formulations [10] and assuming that $\mathbf{a}$ is exact, one obtains an upper and a lower bound for the real coenergy:

$$\mathcal{E}_1^C \geq \mathcal{E}^C \geq \mathcal{E}_2^C. \tag{11}$$

In particular, the equality holds if and only if the constitutive law: $\mathbf{j}_2 = \sigma\mathbf{e}_1$ is exactly fulfilled.

TABLE I
DISCRETE COENERGIES FOR AN ELLIPSOID (μV/m)

Nb of nodes	$\mathcal{E}_1^C$	$\mathcal{E}_2^C$	$\mathcal{E}_1^C - \mathcal{E}_2^C$
250	20.1750	18.4617	1.7133
$3{\cdot}10^3$	20.1197	19.7165	0.4032
$10{\cdot}10^3$	20.1047	19.9801	0.1246
$75{\cdot}10^3$	20.0937	20.0771	0.0166

IV. Exposure to a Homogeneous Field

We simulated the exposure of an homogeneous ellipsoid to a vertical homogeneous 500 μT flux density at 50 Hz. The bound (11) has been tested with progressively refined meshes. The values shown in table I confirm the result.

In order to test the dual formulations with a more realistic model, we considered (under the same exposure conditions) a heterogeneous computational phantom (Fig. 1) based on the Visible Human Project (VHP). The computation has been repeated with a refined model, obtained by splitting each tetrahedral element. The induced electric field computed with the original and refined mesh along a horizontal line at the level of the chest is depicted in Fig. 2. One observes that the error estimates decrease when the mesh is refined. Moreover, the values computed with the refined mesh are nearly always well inside the error bars computed with the original unrefined mesh.

V. Conclusion

The effectiveness of using dual formulations for improving the accuracy of the computation and to estimate the numerical error is demonstrated. On-going works aims at implementing adaptive strategies. In the full paper we will present the application of this method to the study of fields induced by a real device (by using the measured field as source term), as well as the proof of (11).

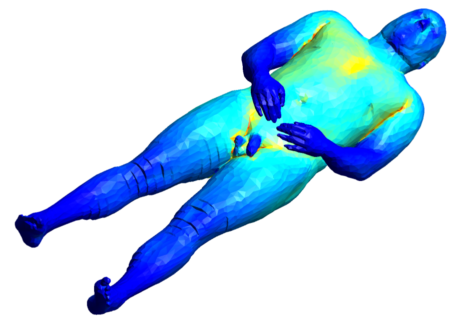

Fig. 1. Electric field on the surface of the phantom from the VHP.

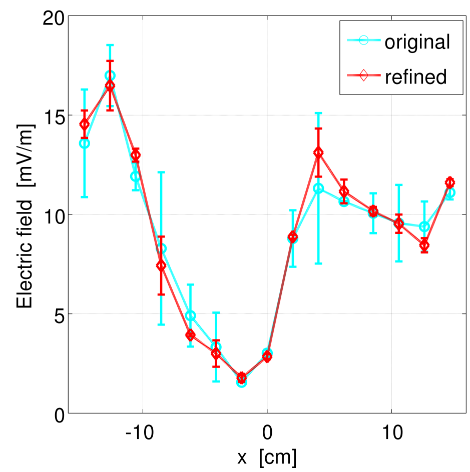

Fig. 2. Electric field computed at the level of the chest.

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